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18EC54

Fifth Semester B.E. Degree Examination, July/August 2021 Information Theory and Coding

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. Define the following with respect to information theory:

(i) Self information	(ii) Entropy	
(iii) Rate of information	(iv) Source efficiency	(04 Marks)
- b. Find the relationship between Hartley's nats and bits. (06 Marks)
- c. Consider the Markov source shown in Fig.Q1(c). Find:

(i) State probabilities	(ii) State entropies	(iii) Source entropy
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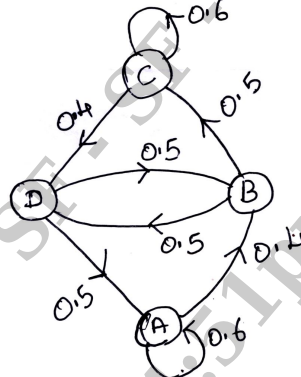


Fig.Q1(c)

(10 Marks)

- 2 a. A source emits one of the four probable messages m_1, m_2, m_3, m_4 with probabilities of $7/16, 5/16, 1/8$ and $1/8$ respectively. Find the entropy of the source. List all the elements for the second extension of this source. Hence show $H(s^2) = 2H(s)$. (08 Marks)
- b. Prove extremal property of entropy. (06 Marks)
- c. In a facsimile transmission of picture, there are about 2.25×10^6 pixel frame. For a good reproduction 12 brightness levels are necessary. Assume all these levels are equally likely to occur. Find the rate of information if one picture is to be transmitted every 3 minutes. What is the source efficiency of this facsimile transmitter? (06 Marks)
- 3 a. Define non-singular and uniquely decidable codes with an example. (04 Marks)
- b. A source emits an independent sequence of symbols from an alphabet consisting of five symbols A, B, C, D and E with probabilities of $1/4, 1/8, 1/8, 3/16$ and $5/16$ respectively. Find the Shannon code for each symbol and efficiency of the coding scheme. (10 Marks)
- c. State and prove Shannon's first theorem. (06 Marks)
- 4 a. State Prefix and Kraft McMillan inequality property. (04 Marks)
- b. A source produces nine symbols x_1, x_2, \dots, x_9 with respective probabilities of 0.24, 0.23, 0.19, 0.13, 0.08, 0.06, 0.04, 0.02 and 0.01.
 - (i) Construct a Shannon-Fano ternary code. (04 Marks)
 - (ii) Determine the code-efficiency and redundancy. (06 Marks)
 - (iii) Draw code-tree. (06 Marks)
 - (iv) Determine the probabilities of 0, 1 and 2 when the encoding alphabet is $\{0, 1, 2\}$. (10 Marks)



c. Find the minimum number of symbols 'r' in the coding alphabet for devising an instantaneous code such that $w = \{0, 5, 0, 5, 5\}$. Devise such a code.
 (Note: w represents the set of code words of length 1, 2, 3....) (06 Marks)

5 a. Show that $H(X, Y) = H\left(\frac{X}{Y}\right) + H(Y)$. (04 Marks)

b. A non-symmetric binary channel is given in Fig.Q5(b).

(i) Find $H(X)$, $H(Y)$, $H\left(\frac{X}{Y}\right)$ and $H\left(\frac{Y}{X}\right)$ given $P(X = 0) = \frac{1}{4}$, $P(X = 1) = \frac{3}{4}$, $\alpha = 0.75$, $\beta = 0.9$.

(ii) Find the capacity of the binary symmetric channel if $\alpha = \beta = 0.75$.

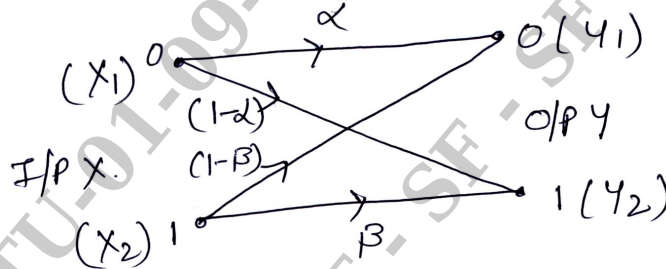


Fig.Q5(b)

c. Show that the mutual information of a discrete channel is symmetric. (06 Marks)

6 a. Derive an expression for channel capacity of binary Erasure channel. (08 Marks)

b. For the JPM given below, compute individually $H(X)$, $H(Y)$, $H(X, Y)$, $H\left(\frac{X}{Y}\right)$, $H\left(\frac{Y}{X}\right)$ and $I(X, Y)$.

$$P(X, Y) = \begin{bmatrix} 0.05 & 0 & 0.20 & 0.05 \\ 0 & 0.10 & 0.10 & 0 \\ 0 & 0 & 0.20 & 0.10 \\ 0.05 & 0.05 & 0 & 0.10 \end{bmatrix}$$
(08 Marks)

c. What is joint probability matrix? State its properties. (04 Marks)

7 a. Define Hamming weight, Hamming distance and minimum distance of linear block codes (with example). (06 Marks)

b. For a systematic (7, 4) linear block code, the parity matrix P is given by

$$[P] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (i) Find G and H.
- (ii) Draw the encoding circuit.
- (iii) Find all possible valid code vectors.
- (iv) A single error has occurred each of these received vectors. Detect and correct those errors. (1) RA = [0111110] (2) RB = [1011100]
- (v) Draw the syndrome calculation circuit. (14 Marks)



- 8 a. The generator polynomial of a (15, 7) cyclic code is given by $g(x) = 1 + x^4 + x^6 + x^7 + x^8$.
- Draw the syndrome calculation circuit.
 - Find the syndrome of the received polynomial $z(x) = 1 + x + x^3 + x^6 + x^8 + x^9 + x^{11} + x^{14}$ by listing the states of the register used in syndrome calculation circuit.
 - Verify the syndrome obtained in (ii) by using direct hand calculation. **(10 Marks)**
- b. Consider the (15, 11) cyclic code generated by $g(x) = 1 + x + x^4$.
- Draw the feedback register encoding circuit for this cyclic code.
 - Illustrate the encoding procedure with the message vector 01101001011 by listing the state of the register with each input.
 - Verify the code polynomial by using the division method. **(10 Marks)**
- 9 a. What are convolutional codes? How it is different from block codes. **(05 Marks)**
- b. Consider the convolutional encodes shown in Fig.Q9(b).
- Find the O/P for the message 10011 using time domain approach.
 - Find the O/P for the message 10011 using transform domain approach.

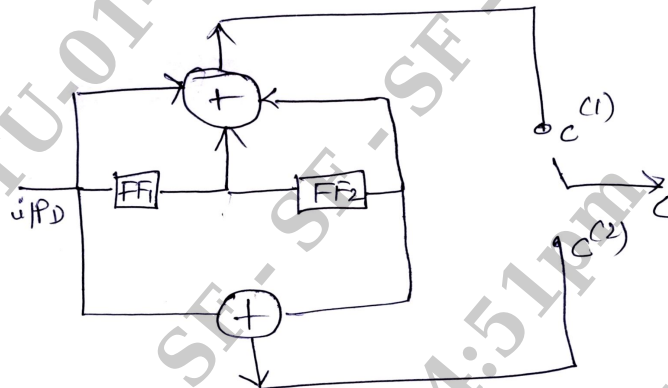


Fig.Q9(b)

- c. What do you understand by trellis diagram of a convolutional encodes? Explain clearly. **(05 Marks)**
- 10 a. For (2, 1, 3) convolution encodes with $g(1) = 1011$, $g(2) = 1101$.
- Write translation table.
 - State diagram.
 - Draw the code tree.
 - Draw the trellis diagram.
 - Find the encoded O/P for the message 11101 by traversing the code tree. **(15 Marks)**
- b. Explain Viterbi encoding. **(05 Marks)**
